GCSE Maths – Algebra

Functions

Worksheet

NOTES

SOLUTIONS

This worksheet will show you how to work out different types of function questions. Each section contains a worked example, a question with hints and then questions for you to work through on your own.

Section A

Worked Example

Find the value of $g(x) = x^3 + 2x^2 - 9x + 1$ when $x = 3$.

Step 1: Substitute $x = 3$ into the function $g(x)$. Replace every x in the function with the numerical value 3.

$$g(3) = (3)^3 + 2(3)^2 - 9(3) + 1$$

Step 2: Simplify where possible by multiplying out the brackets and applying powers.

$$27 + 2(9) - 27 + 1 = 27 + 18 - 27 + 1$$

Step 3: Complete the final sum.

$$27 + 18 - 27 + 1 = 18 + 1 = 19$$

Answer: $g(3) = 19$

Guided Example

If $f(x) = (x + 7)(6 - x^2)(4 + x)$ what is the value of $f(4)$?

Step 1: Substitute $x = 4$ into the function $f(x)$.

Step 2: Simplify each term where possible.

Step 3: Compute the final sum.

1. Complete the following:

a) Find the value of $f(5)$ when $f(x) = 3x - 2$.

b) Find the value of $g(7)$ when $g(x) = 4x - x^2$.

c) Find the value of $h(9)$ when $h(a) = \sqrt{a} - 2a^3 + 1$.

d) Find the value of $f(-8)$ when $f(x) = 5x + 3(\sqrt[3]{x}) - 2x^2$.

2. Solve the following for x:

a) $h(x) = x^2 - 5x + 2$, $h(x) = -4$.

b) $f(x) = 18x - 2$, $f(x) = 6$.

3. If $g(x) = x^3 - 2(\sqrt[4]{x}) + 1$ what is an expression for $g(16x)$?

Section B – Higher Only

Worked Example

Find the inverse function of $f(x) = 2x + 1$.

Step 1: Replace $f(x)$ with y.

$$y = 2x + 1$$

Step 2: Rearrange the terms to make x the subject of the equation.

$$y - 1 = 2x$$

$$\frac{y - 1}{2} = x$$

Step 3: Replace x with y and y with $f^{-1}(x)$.

$$\frac{x - 1}{2} = f^{-1}(x)$$

Answer: $f^{-1}(x) = \frac{x-1}{2}$

Guided Example

Find the inverse function of $h(x) = \frac{2-x}{x}$.

Step 1: Replace $h(x)$ with y.

Step 2: Make x the subject of the equation.

Step 3: Replace x with y and y with $h^{-1}(x)$.

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4. Find the inverse of the following functions:

a) $f(x) = 4x + 7$

b) $g(x) = 15x^2 + 3$

c) $f(x) = \frac{2x}{3+x}$

d) $h(x) = \frac{4-3x}{x+3}$

5. Find the value of $f(x) = x^2 - 3$ if $x = f^{-1}(6)$.

6. Find the value of $g(x) = \frac{9-9x^2}{x^2}$ if $x = g^{-1}(7)$.

7. Solve $h(x) = 3x^2 + 6$ for x given $h^{-1}(x) = 2$.

8. If $f^{-1}(x) = 5x^2 - 3$ what is $f(x)$?

Worked Example

Given $f(x) = 2x - 7$ and $g(x) = 4x + 5$ find $fg(x)$.

Step 1: Place the innermost function $g(x)$ into the outermost function $f(x)$ in place of x.

$$fg(x) = 2g(x) - 7 = 2(4x + 5) - 7$$

Step 2: Simplify by expanding the brackets and collecting like terms.

$$fg(x) = 2(4x + 5) - 7 = 8x + 10 - 7$$

$$fg(x) = 8x + 3$$

Answer: $fg(x) = 8x + 3$

Worked Example

Given $f(x) = x^2 - 7x$ and $g(x) = 3x - 3$ find $fg(2)$.

Step 1: Place the innermost function $g(x)$ into the outermost function $f(x)$ in place of x.

$$fg(x) = (3x - 3)^2 - 7(3x - 3)$$

Step 2: Simplify by expanding the brackets and collecting like terms.

$$fg(x) = (3x - 3)(3x - 3) - 7(3x - 3)$$

$$fg(x) = (9x^2 - 9x - 9x + 9) - 7(3x - 3)$$

$$fg(x) = 9x^2 - 18x + 9 - 21x + 21 = 9x^2 - 39x + 30$$

Step 3: Substitute $x = 2$ into $fg(x)$ to find $fg(2)$.

$$fg(2) = 9(2)^2 - 39(2) + 30$$

$$fg(2) = -12$$

Answer: $fg(2) = -12$

Guided Example

Given $f(x) = 5x + 3$ and $g(x) = 3x^2 + 12$, find $gf(x)$.

Step 1: Place the innermost function $f(x)$ into the outermost function $g(x)$ in place of x.

Step 2: Simplify by expanding the brackets and collecting like terms.

Guided Example

Given $f(x) = \dfrac{x^2-3}{x}$ and $g(x) = 3x^2 + 12$, find $fg(6)$.

Step 1: Place the innermost function $g(x)$ into the outermost function $f(x)$ in place of x.

Step 2: Simplify by expanding the brackets and collecting like terms.

Step 3: Substitute $x = 6$ into $fg(x)$ to find $fg(6)$.

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

8. Find $fg(x)$, given $f(x) = 3(x - 4)$ and $g(x) = \frac{x}{5} + 1$.

9. Find $gf(x)$, given $g(x) = 3x + 5$ and $f(x) = \frac{1}{3}x - \frac{5}{2}$.

10. Given $f(x) = 1 - 2x^3$ and $g(x) = \frac{3}{x} - 4$, show that $gf(x) = \frac{8x^3 - 1}{1 - 2x^3}$.

11. Given $f(x) = 4x + 6$ and $g(x) = x^2 - 9$, find the value of $fg(3)$.

12. Given $f(x) = x^2 - 4$ and $g(x) = 4x - 1$, find the value of $fg(x - 2)$.

13. Given $g(x) = \frac{5}{4x}$ and $f(x) = 7x^2 + 3x - 2$, find the value of $gf(4)$.

14. If $f(x) = x^2 + 1$, what is $ff(x)$?

15. If $f(x) = 2 - 3x^2$ and $g(x) = 12x - 1$, which value is greater: $fg(3)$ or $gf(3)$?